Manifold Sampling for Nonconvex Piecewise Continuously Differentiable Functions

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Problem Statement

We are interested in solving the problem:

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad f(x) \equiv h(F(x))$$

where $F : \mathbb{R}^n \to \mathbb{R}^m$, $h : \mathbb{R}^m \to \mathbb{R}$,

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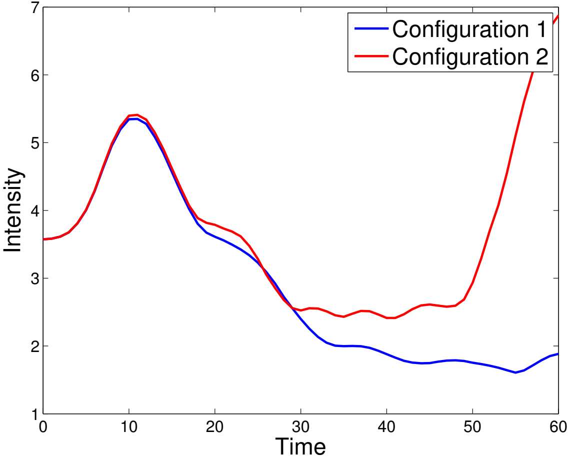
Though h is piecewise linear, F being nonlinear implies f can be nonlinear

Laser pulse propagating in a plasma channel

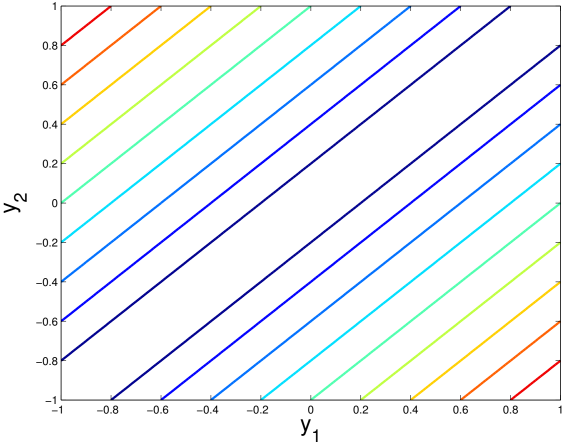
Want to determine the plasma channel properties so the maximum difference in the laser intensity during propagation is minimized.



$$f(x) = \max\{F_i(x)\} - \min\{F_i(x)\}$$

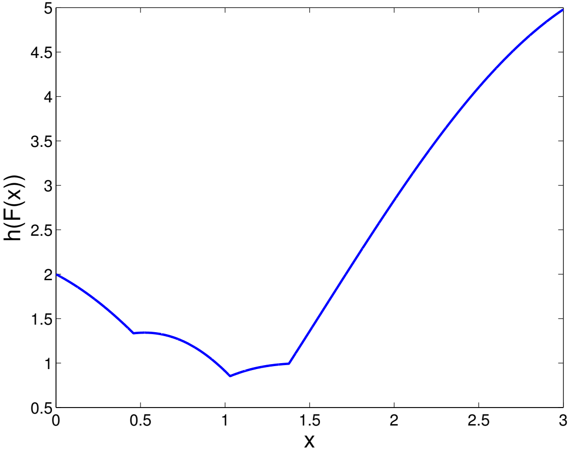
Formulation

$$h(y) = \max\{y\} - \min\{y\}$$



Formulation

$$h(F(x)) = \max\{\sin(2x)+1, \cos(2x), x\} - \min\{\sin(2x)+1, \cos(2x), x\}$$



Nonsmooth Optimization

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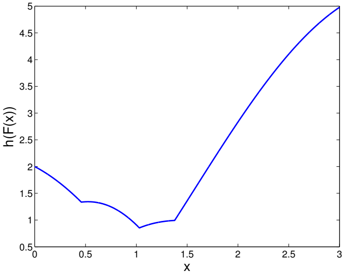

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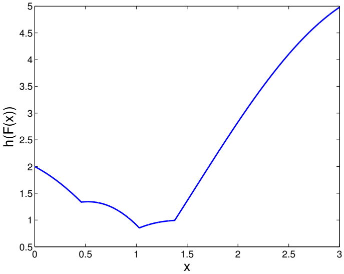

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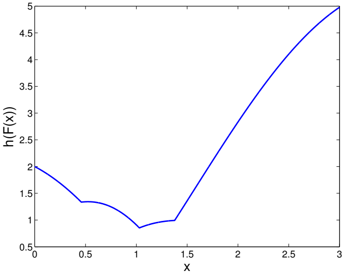

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Just running a smooth algorithm on a nonsmooth problem may not converge, or may converge to a nonstationary point.

A generalized derivative

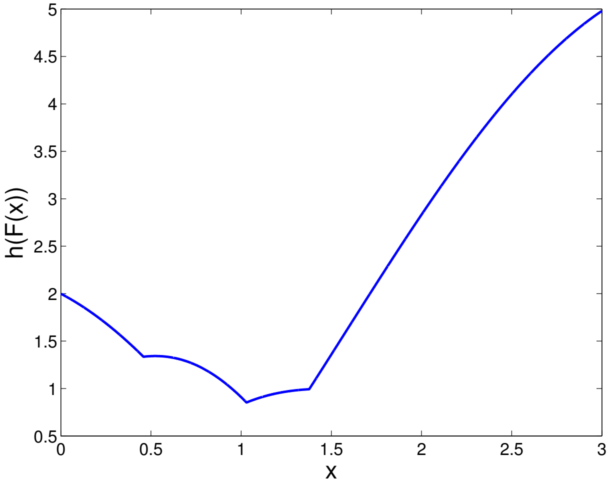
Definition

For locally Lipschitz continuous functions $f : \mathbb{R}^n \to \mathbb{R}$, the *generalized Clarke subgradient* of f at a point $x \in \mathbb{R}^n$ is:

$$\partial f(x) = \operatorname{conv}\left\{\xi \in \mathbb{R}^n : \xi = \lim_{x^i \to x} \nabla f(x^i) \text{ and } \nabla f(x^i) \text{ exists at all } x^i\right\}$$

where $\operatorname{conv}(\cdot)$ denotes the convex hull of a set.

A generalized derivative



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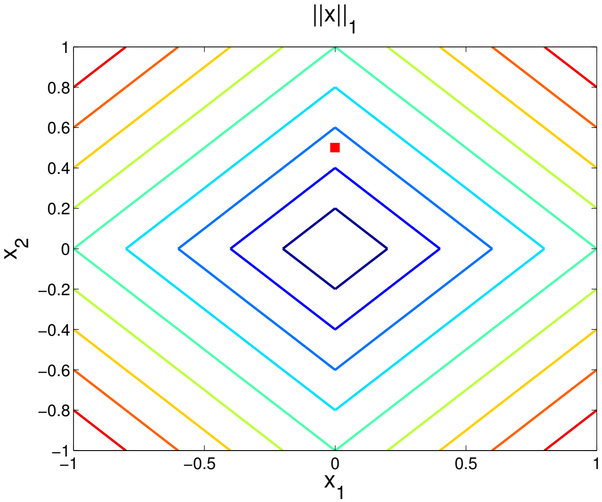

$$\partial \left\| [0, 0.5] \right\|_1 = \operatorname{conv} \{ [1, 1], [-1, 1] \}$$

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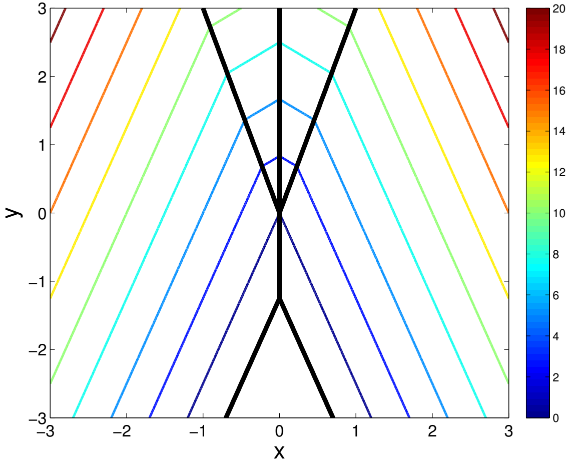
- If f is locally Lipschitz continuous and continuously differentiable at x, $\nabla f(x) = \partial f(x)$.
- $0 \in \partial f(x)$ is a necessary optimality condition for locally Lipschitz continuous f.

Steepest descent can have trouble. . .

$$f(x,y) = \max\left\{-\frac{5}{2}, \pm 2x + 3y, \pm 5x + 2y\right\}$$

Piecewise affine, convex function.

Steepest descent can have trouble...



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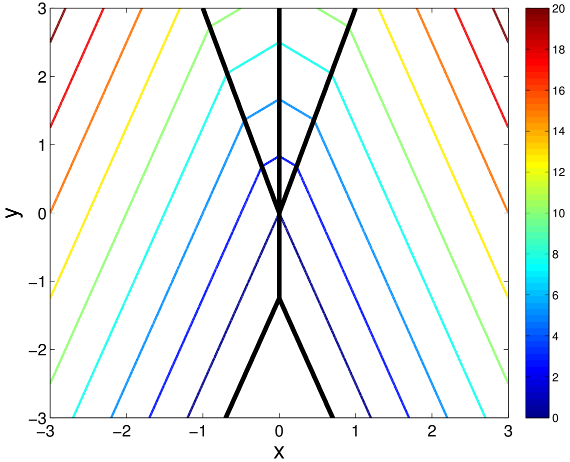

Steepest descent with exact line search started from any point on the diagonal lines will converge to $(0, 0)$.

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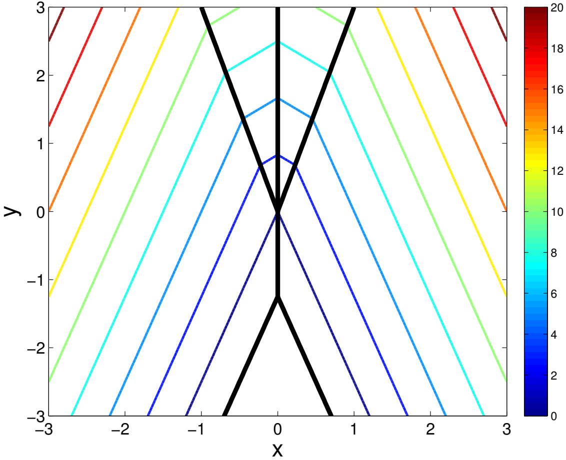

Steepest descent with exact line search started from any point above the diagonal lines will converge to $(0, 0)$.

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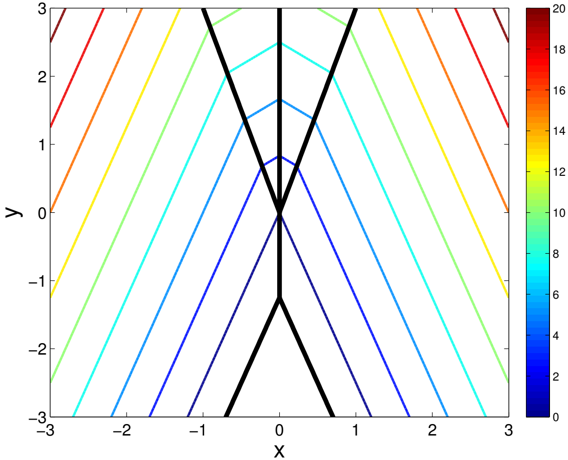

Steepest descent with inexact line search (producing points close to the diagonal) started from any point above the diagonal lines will converge to $(0, 0)$.

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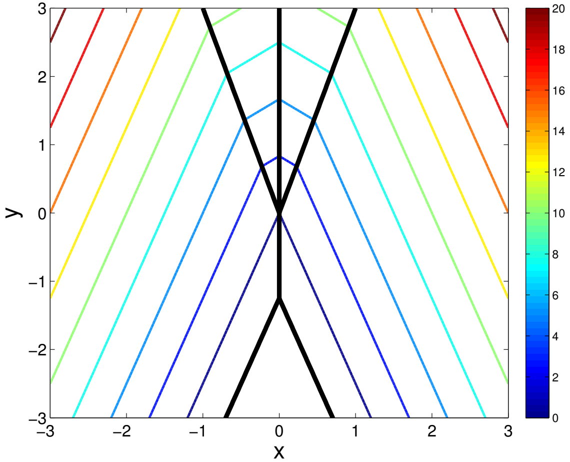

Descent (with directions sufficiently close to steepest) with inexact line search (producing points close to the diagonal) started from any point above the diagonal lines will converge to $(0, 0)$.

Subgradient methods

Approaches for nonsmooth optimization

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- Not a descent method because the step sizes are fixed.
- Not a descent method because ξ^k may not be a descent direction.

Gradient Sampling

Approaches for nonsmooth optimization

Theorem (Rademacher)

If $S \subset \mathbb{R}^n$ is open and $f : S \to \mathbb{R}$ is locally Lipschitz on S, then f is differentiable almost everywhere on S.

Gradient Sampling

1. Approximate $\partial f(x^k)$ by sampling $m \geq n+1$ points $x^{k,j}$ in $\mathcal{B}(x^k, \epsilon_k)$. Set

$$G^k = \text{conv}\left\{\nabla f(x^{k,1}), \dots, \nabla f(x^{k,m})\right\}$$

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   Else, find a point in $\hat{x} \in \mathcal{B}(x^k, \epsilon_k)$ satisfying
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- Iterates must not be at points of nondifferentiability
- A lot of sampling may be required

Trust region methods

Smooth case

1. Build a model m_k of f at x^k, for example

$$m_k(p) = f(x^k) + \nabla f(x^k)^T p + \frac{1}{2} p^T \nabla^2 f(x^k) p$$

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4. If $\rho_k > \eta > 0$, $x^{k+1} = x^k + s^k$, $\Delta_{k+1} = \gamma_{\mathrm{inc}} \Delta_k$.
   Else $x^{k+1} = x^k$, $\Delta_{k+1} = \gamma_{\mathrm{dec}} \Delta_k$.

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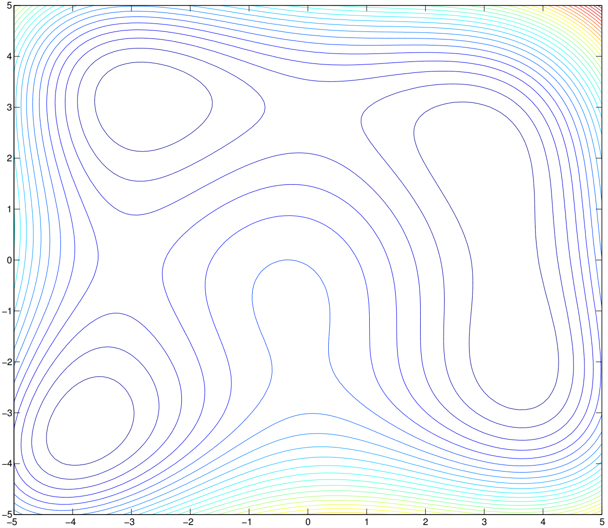

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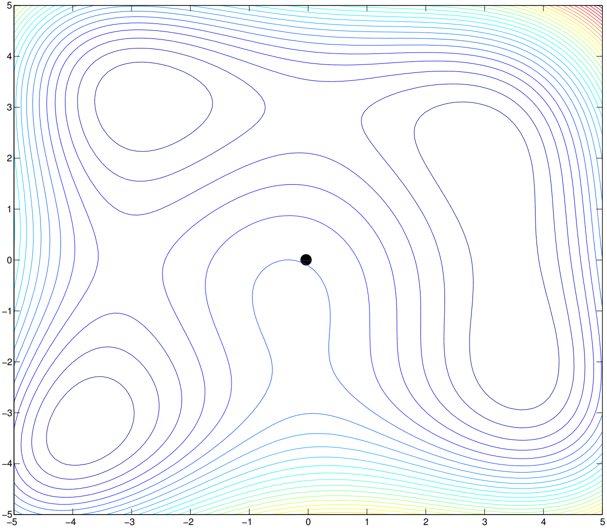

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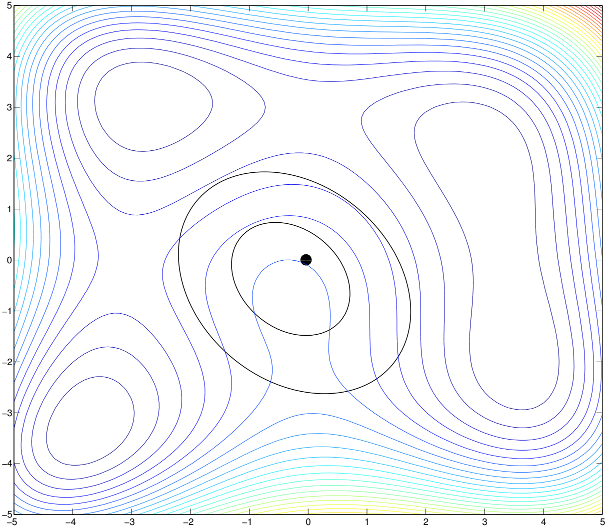

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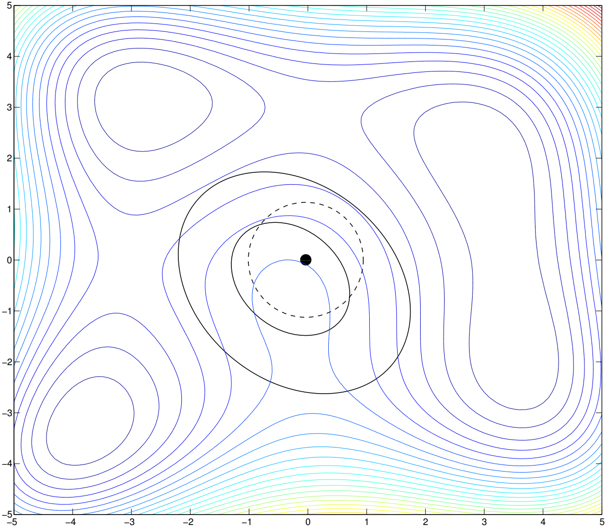

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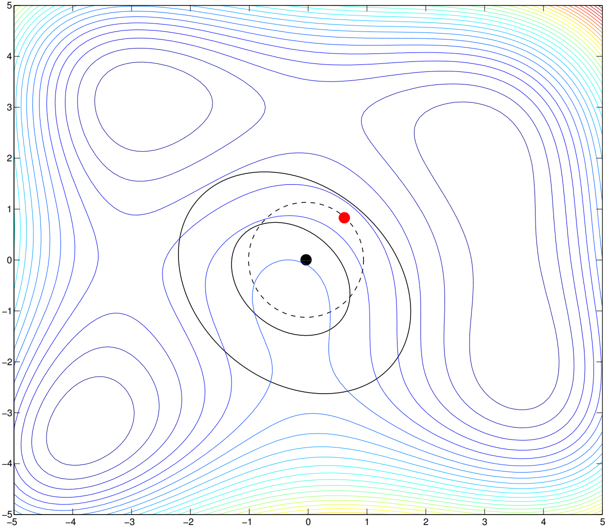

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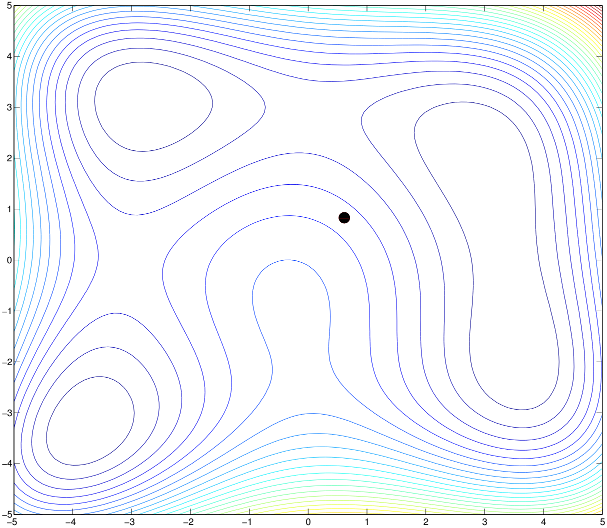

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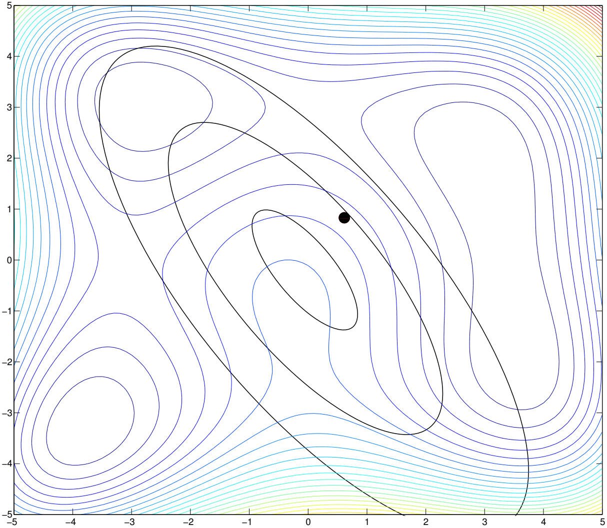

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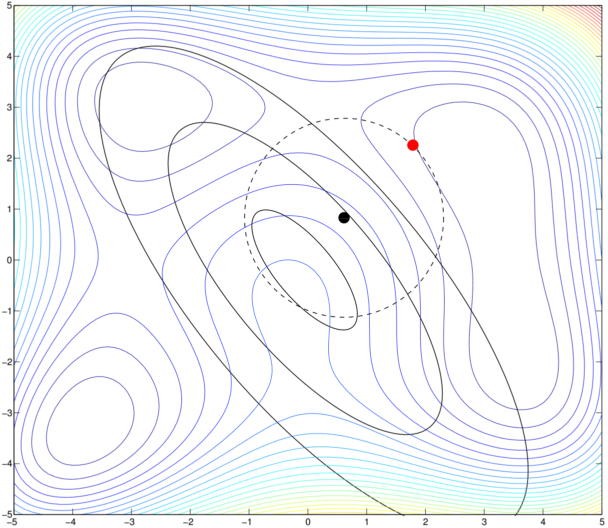

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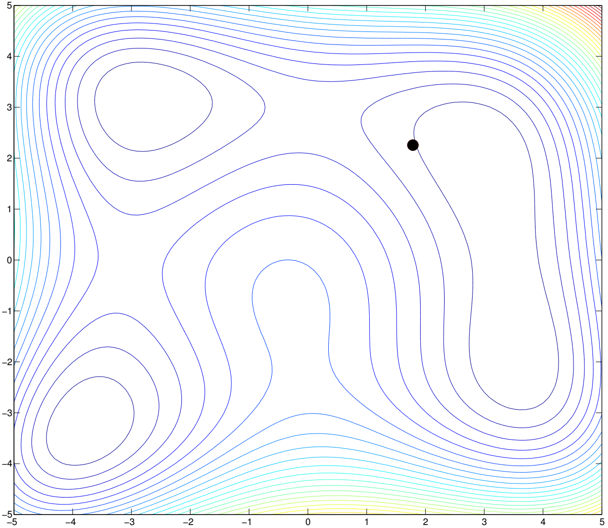

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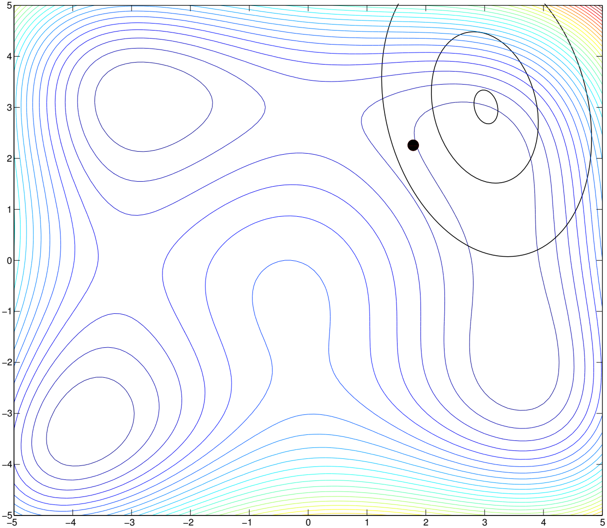

Traditional ρ test

Smooth Case

A model m sufficiently approximates f near x if

$$\begin{aligned} |f(x+s) - m(x+s)| &\le c_1 \Delta^2 \ \forall s \in \mathcal{B}(0,\Delta) \\ \|\nabla f(x+s) - \nabla m(x+s)\| &\le c_2 \Delta \ \forall s \in \mathcal{B}(0,\Delta), \end{aligned}$$

with c_1 and c_2 independent of Δ and x.

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We want

$$\left|\frac{f(x^k) - f(x^k+s^k)}{m(x^k) - m(x^k+s^k)} - 1\right| \leq c\Delta_k$$

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We want

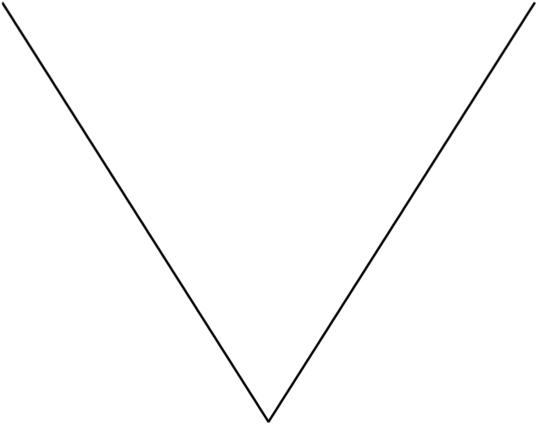
$$\left|\frac{f(x^k) - m(x^k) + m(x^k + s^k) - f(x^k + s^k)}{m(x^k) - m(x^k + s^k)}\right| \approx \frac{c_1\Delta_k^2 + c_1\Delta_k^2}{\|\nabla m(x)\|\,\Delta_k} \approx c_3\Delta_k$$

A new ρ test

Composite nonsmooth case

For nonsmooth functions, we do not get this.
If x^k and $x^k + s^k$ are on different sides of the absolute value kink,

$$\|\nabla f(x+s) - \nabla m(x+s)\| = 2$$



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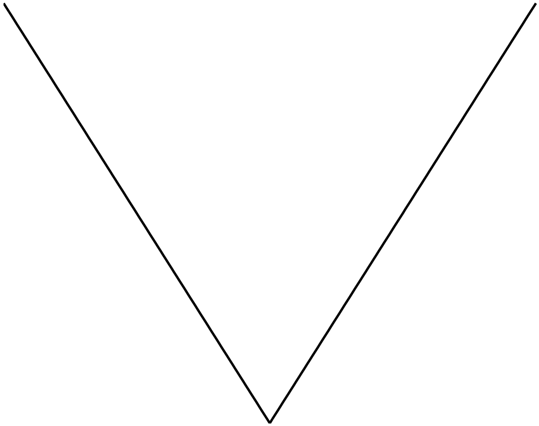
But if we include information at both x^k and $x^k + s^k$ when deciding on s^k, then we can ensure our model accurately approximates f on $\operatorname{conv}\left\{x^k, x^k + s^k\right\}$.



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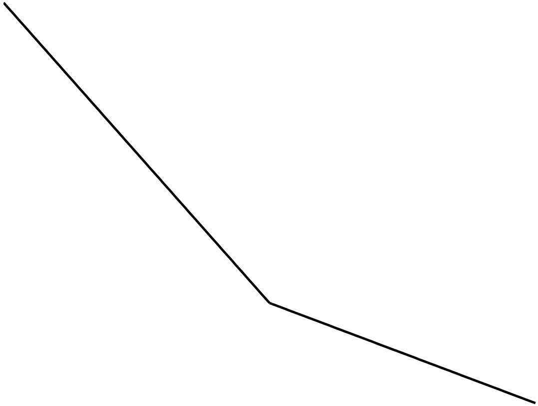

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Composite nonsmooth case

Let h_x and h_s be the affine functions active at $F(x^k)$ and $F(x^k + s^k)$, respectively. Then there are three cases.

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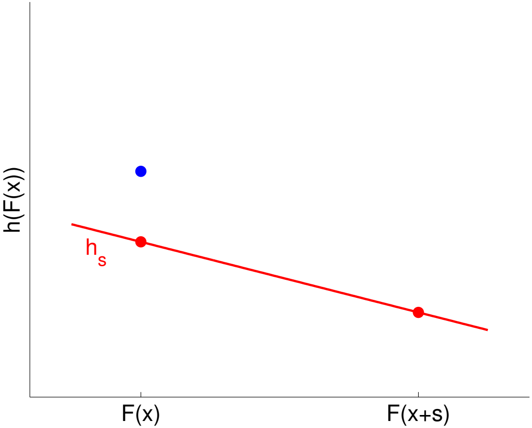

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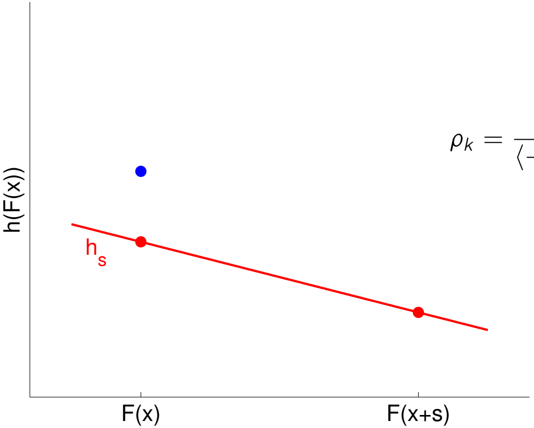

$$\rho_k = \frac{h_s(F(x^k)) - f(x^k + s^k)}{\langle -s^k, \nabla M(x^k) \nabla h_s(F(x^k)) \rangle}.$$

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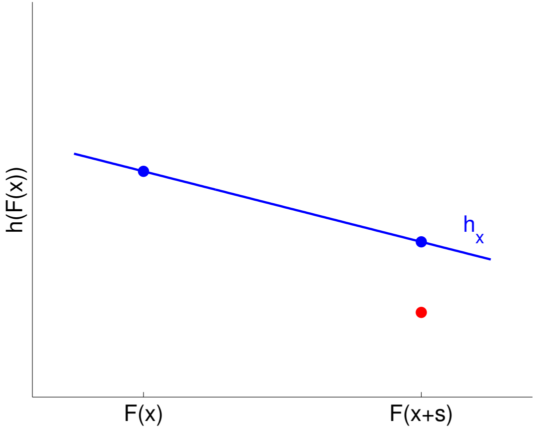

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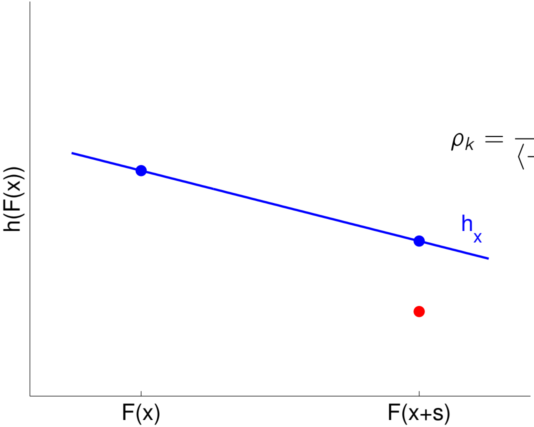

$$\rho_k = \frac{f(x^k) - h_x(F(x^k + s^k))}{\langle -s^k, \nabla M(x^k) \nabla h_x(F(x^k)) \rangle}.$$

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Let h_x and h_s be the affine functions active at $F(x^k)$ and $F(x^k + s^k)$, respectively. Then there are three cases.

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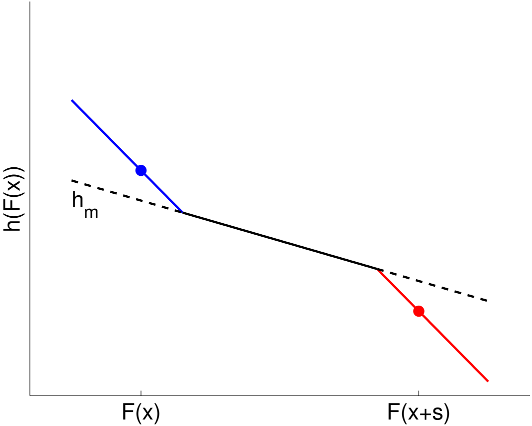

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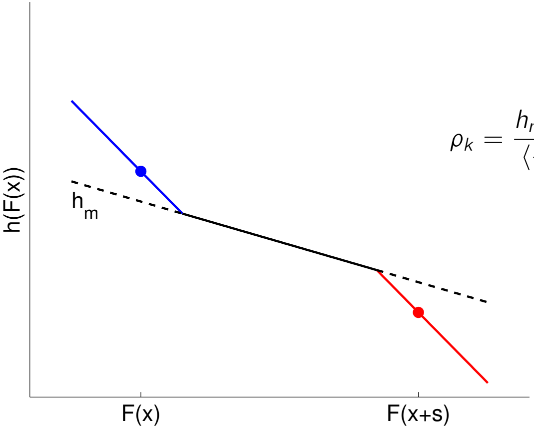

$$\rho_k = \frac{h_m(F(x^k)) - h_m(F(x^k + s^k))}{\langle -s^k, \nabla M(x^k) \nabla h_m(F(x^k)) \rangle}.$$

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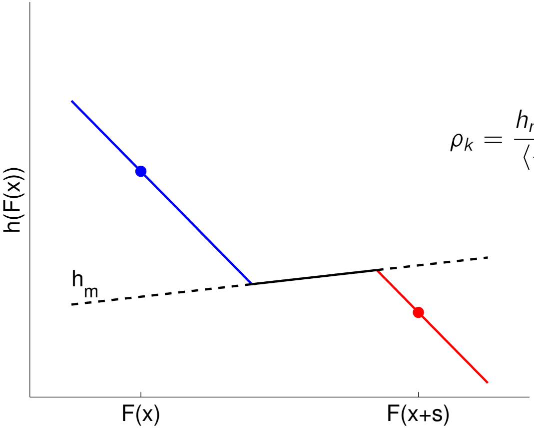

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Trust region methods

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1. Build a model $m_k^{F_i}$ of F_i at x^k.

Trust region methods

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2. Use ∇m^{F_i} to form $\nabla M(x)$ and build a set of generators G^k. Set ξ^k to be the minimum norm element in G^k.

Generator set

$$G^k = \bigcup_{i \in I_h(F(x^k))} \{\nabla M(x^k) \nabla h_i(F(x^k))\}$$

where $I_h(F(x^k))$ is the set of indices for the piecewise affine parts h_i that define h and that are active at $F(x^k)$.

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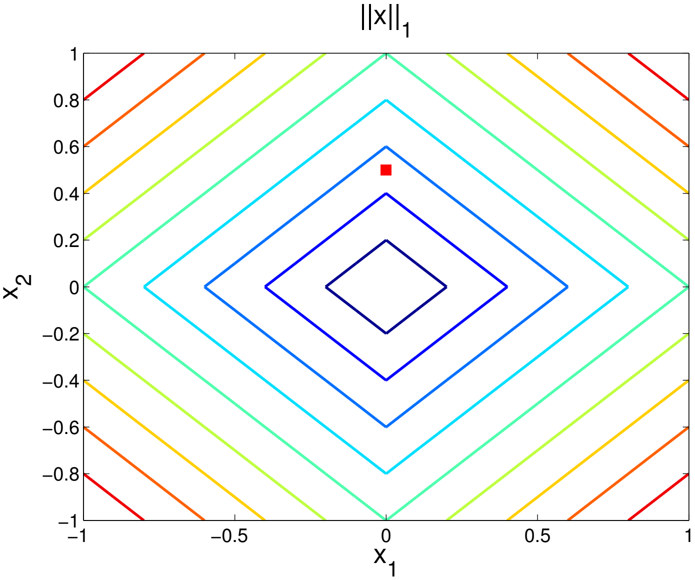
Or, given a set of points $Y = \left\{x^k, y^2, \ldots, y^p\right\} \subset \mathcal{B}(x^k, \Delta_k)$,

$$G_k = \bigcup_{y \in Y} \bigcup_{i \in I_h(F(y))} \left\{ \nabla M(x^k) \nabla h_i(F(y)) \right\}$$

Generator set



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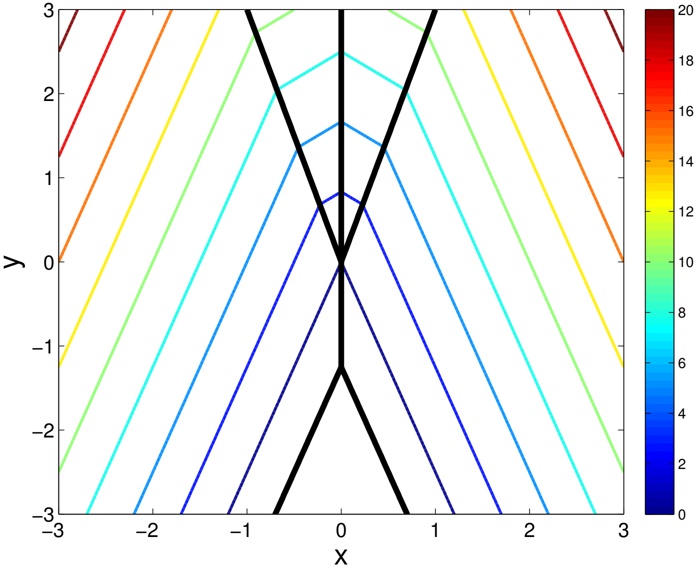

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4. Ensure the correct manifolds are in G^k, depending on the case. Either add them to G^k and go to 2, or calculate ρ_k.
5. If $\rho_k > \eta > 0$, $x^{k+1} = x^k + s^k$, $\Delta_{k+1} = \gamma_{\mathrm{inc}}\Delta_k$.
   Else $x^{k+1} = x^k$, $\Delta_{k+1} = \gamma_{\mathrm{dec}}\Delta_k$.

Conclusions

- Nonsmooth problems appear in many places and motivate active optimization research.
  See: **MS76** Wed. 10:30 AM – 12:30 PM, BCEC Room 253C:
  Sensitivity Analysis and Optimality Conditions in Nonsmooth Problems.

Conclusions

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  See: **MS76** Wed. 10:30 AM – 12:30 PM, BCEC Room 253C: Sensitivity Analysis and Optimality Conditions in Nonsmooth Problems.

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Conclusions

- Nonsmooth problems appear in many places and motivate active optimization research.
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- Functions of the form $h(F(x))$ with piecewise linear h encompass a large class of nonsmooth f.

- Often, the nonsmoothness has a known form. Exploiting this can be beneficial to performance.
  Larson, Menickelly, Wild. "Manifold Sampling for L1 Nonconvex Optimization."
  See: **IT5** Wed. 8:30 AM – 9:15 AM, Grand Ballroom: Beyond the Black Box in Derivative-Free and Simulation Based Optimization.